Professional Development Module

Title: Teaching Fractions in Grades 3 - 6

Content and Instructional Shifts: K-5

Targeted Audience: Teachers in grades 3-6

Grade Span: 3-6

Description: Instructor notes; handouts; implementation assignments – based on *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

Delivery time: Session 3 of 10 three-hour sessions

The following materials were designed with the intent that the presenter(s) would be educators who have a deep understanding of the mathematical content being addressed at this level.

Iowa Core Mathematics: Teaching Fractions in Grades 3-6
Session 3 Instructor Notes

Learning Goals:

- Teachers will understand the content and instructional shifts for teaching fractions resulting from adoption of *Iowa Core Mathematics.*
- Teachers will understand the grade-specific expectations and cross grade-level learning progressions of the *Iowa Core Mathematics* fraction standards.
- Teachers will understand and implement research-based instructional strategies to build students' understanding of fractions and algebra.

Success Criteria:

- Teachers will classify fraction word problems according to the problem situations described in Table 2 of *Iowa Core Mathematics*.
- Teachers will identify the common strategies for solving Multiple Groups problems (see *Extending Children's Mathematics* p.62).
- Teachers will describe the importance of the number line diagram in *Iowa Core Mathematics* and explain ways to help students use this model.

Time: 3 hours

Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Iowa Core Mathematics Content and Practice Shifts Grades K-5"
- Handout "Multiple Groups Problem Situations"
- Handout "Iowa Core Mathematics Tables 1 & 2 Problem Situations"
- Handout "Iowa Core Mathematics Measurement and Data Standards"
- Handout "Session 3 Assignment Sheet"
- Student work collected by each participant
- Black fine point markers (2-3 per group)
- Unlined $8\frac{1}{2}$ by 11-inch paper (5-6 per group)

Session 3 Activity 1
Analyze Content Shifts Document

Approximate Time: 15 minutes
Key Purpose: To review the first three fraction shifts and connect the shifts to previous classroom learning.
Materials:

- Handout "Iowa Core Mathematics Content and Practice Shifts Grades K-5"

Activity Description	Key Discussion Points
Content and Practice Shifts Participants read the first three shifts, pp. 9-11 of "Iowa Core Mathematics Content and Practice Shifts Grades K-5." Pose the following questions: What stands out in these shifts? What is new or unique? Is there anything you question or find troublesome? Why does *Iowa Core Mathematics* emphasize the number line in the fraction domain? How do Equal Sharing problems connect to these shifts?	**Content and Practice Shifts** • In order for students to make sense of fractions and fraction operations, they must understand the relationship between the unit fraction and composite fraction. Equal Sharing problems help students develop this understanding. • *Iowa Core Mathematics* emphasizes the number line model. The number line is more than an additional strategy for solving problems. The number line helps students understand a fraction as a quantity. (This connects back to the misconception of a fraction being two whole numbers.) • The number line does not fit the context of the brownie problem. The ribbon problem described in the shifts document is an equal sharing problem that lends itself to a number line model. • As students are expected to use the number line diagram for fractions, starting in 3rd grade, teachers should give students multiple opportunities to solve problems that lend themselves to this representation. We will spend time discussing number lines during this session.

Session 3 Activity 2
Analyze Student Work from Implementation Assignment 2

Approximate Time: 45 minutes
Key Purpose: To analyze student work and reflect on classroom experiences.
Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Student work collected by each participant

Small Group Work Place participants in groups of three to four teachers. Have each participant share how they classified their student work and give	**Small Group Work** • The purpose of this activity is for teachers to:

examples of the different categories of strategies their students used. Also have participants answers the following questions: • Do you think the new problem was easier or more difficult than the brownie problem? • How are the strategies your students used for the new problem similar or different from the strategies your students used for the brownie problem? • Are there students who struggled with the new problem who were able to solve the brownie problem? Are there students who struggled with the brownie problem who were able to solve the new problem? Why do you think this happened? • What are your next steps? What problem will you give next? Give time for each group to share highlights from their discussion with the entire group.	○ Check their categorization of student work. ○ Discuss the impact of number choice on students' strategies. The number choice often makes a problem more or less difficult. ○ Plan next steps based on student understanding. Teachers make instructional decisions to further develop student understanding based on what their students understand as shown in the strategies students use. • We hope teachers will discuss specific number choices such as: ○ Some numbers allow for multiple equivalent fractions. ○ Problems resulting in unit fractions tend to be easier for students. For example, 6 sharing 7 tends to be easier to solve than 6 sharing 8. • We also hope teachers will discuss their next steps and the rationale for their decision. Discussion time is limited, so this should be an ongoing discussion throughout the class.

Session 3 Activity 3
Multiple Groups Problems

Approximate Time: 50 minutes
Key Purpose: To recognize the differences among multiplication, partitive division, and measurement division problems involving fractions.
Materials:
- Handout "Multiple Groups Problem Situations"
- Handout "Iowa Core Mathematics Tables 1 & 2 Problem Situations"

Activity Description	Key Discussion Points
1. " Multiple Groups Problem Situations" Questions 1 & 2 Have the participants complete "Multiple Groups Problem Situations" (handout) problems 1 and 2 in small groups. As a large group discuss how participants' diagrams and thinking varied with each problem. Also discuss the equations children might use to represent each problem. • I am making 4 large pizzas and I have 3 pounds of cheese. If I want each pizza to have the same amount of cheese, how much cheese should I put on each pizza? • I am making 4 large pizzas. How much cheese do I need if I want to put $\frac{3}{4}$ pound of cheese on each? • I have 3 pounds of cheese. I want to put $\frac{3}{4}$ pound on each pizza I	**1. " Multiple Groups Problem Situations " Questions 1 & 2** • Participants should recognize the first problem is a Multiple Groups Equal Sharing problem. You know the total (3 pounds of cheese) and the number of groups (4 pizzas). You need to find the size of each group. The 3 pounds of cheese is being equally shared among the 4 pizzas. Children might use a division or multiplication equation such as 3 ÷ 4 = ? or 4 x ? = 3 to represent the situation. • The second problem is a multiplication problem. Both the number of groups (4 pizzas) and size of each group ($\frac{3}{4}$ pound) are known. You need to find the total amount of cheese. • The third problem tells you the total (3 pounds of cheese) and the size of each group (¾ pound). You need to find the number of

make. How many pizzas can I make?

groups. Children might use a division or multiplication equation such as $3 \div \frac{3}{4} = ?$ or $? \times \frac{3}{4} = 3$ to represent the situation.

- The similarity among the three problems is the number sentence. The number sentence $4 \times \frac{3}{4} = 3$ is true for all three problems.
- The major difference among the problems is what is unknown.
- It is important to emphasize the convention in textbooks in the United States is the first factor represents the number of groups and the second factor represents the size of the groups in a multiplication grouping problem.

2. " Multiple Groups Problem Situations " Questions 3 & 4

Have the participants complete problems 3 and 4 in small groups and refer to "Iowa Core Mathematics Tables 1 & 2 Problem Situations" (handout). Discuss the results as a large group.

3. " Multiple Groups Problem Situations " Questions 3 & 4

- While the problems are similar, the unknown is different for each problem.

Word Problem	Number of Groups	Size of Group	Total	Possible Equation(s)	Problem Type
I am making 4 large pizzas and I have 3 pounds of cheese. If I want each pizza to have the same amount of cheese, how much cheese should I put on each pizza?	4	unknown	3 pounds	$3 \div 4 = ?$ Or $4 \times ? = 3$	Multiple Groups Partitive Division (Equal Sharing)
I am making 4 large pizzas. How much cheese do I need if I want to put $\frac{3}{4}$ pound of cheese on each?	4	$\frac{3}{4}$ pound	unknown	$4 \times \frac{3}{4} = ?$	Multiple Groups Multiplication
I have 3 pounds of cheese. I want to put $\frac{3}{4}$ pound on each pizza I make. How many pizzas can I make?	unknown	$\frac{3}{4}$ pound	3 pounds	$3 \div \frac{3}{4} = ?$ Or $? \times \frac{3}{4} = 3$	Multiple Groups Measurement Division

Adapted from: Empson, S. B. and Levi, L. (2011). *Extending Children's Mathematics: Fractions & Decimals, Innovations in Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

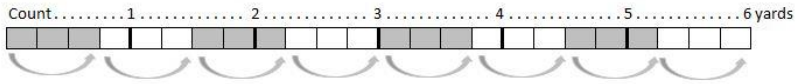
- A division problem tells you the total and either the number of groups or the size of each group. If you know the number of groups, but not the size of each group, it is called a partitive division problem. If you know the size of each group, but not the number of groups, it is called a measurement division problem.
- The three problems are Multiple Groups problems. A Multiple Group problem is one in which there is a whole number of groups

	and a fractional amount in each group where the fraction is not equal to a whole number. An Equal Sharing problem is one type of Multiple Group problem. The problems are also Equal Groups problems. An Equal Groups problem is a problem situation and matches row one of *Iowa Core Mathematics* Table 2. • Note to Instructor: At this point we are only working with Equal Groups situations from Table 2 of *Iowa Core Mathematics*. The problem situations in rows 2 and 3 (Arrays/Area and Comparison problems) will be addressed later in the course.

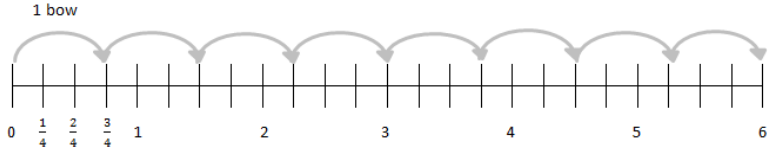
Session 3 Activity 4
Strategies for Multiple Group Problems

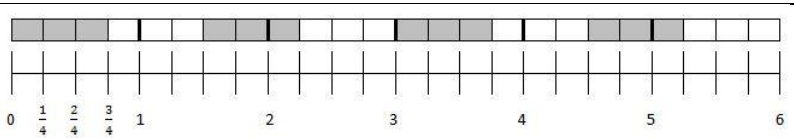
Approximate Time: 60 minutes
Key Purpose: To familiarize teachers with strategies for Multiple Groups problems and the importance of the number line in *Iowa Core Mathematics*.
Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Iowa Core Mathematics Measurement and Data Standards"
- Black fine point markers (2-3 per group)
- Unlined $8\frac{1}{2}$ by 11-inch paper (5-6 per group)

Activity Description	Key Discussion Points
1. Ribbon Problem It takes $\frac{3}{4}$ yard of ribbon to make a bow. How many yards of ribbon will it take to make 8 bows? Ask each small group to find 2 to 3 different ways students might solve the ribbon problem and show each strategy on a different unlined sheet of paper using a black marker. If participants do not generate a variety of strategies consider asking them to think about children at a variety of grade levels. Compare and contrast the strategies as an entire class. Start the discussion by asking groups to share their least sophisticated strategy. After groups share all of their strategies have them turn to page 62 of *Extending Children's Mathematics* and classify the strategies.	**1. Ribbon Problem** This discussion is an introduction to strategies for solving Multiple Groups problems. Participants see additional examples when they read chapter 3 of *Extending Children's Mathematics* after this session. There is limited number of ways to solve a Multiple Groups problem. The following examples show four common ways to solve the ribbon problem. These strategies are described on page 62 of *Extending Children's Mathematics*. Participants' drawings may look different, but reflect the same thinking. If participants do not show all of the strategies listed, you may want to share the additional methods. • Represents Each Group by Direct Modeling:  • Represents Each Group with Repeated Addition:

	$\frac{3}{4}+\frac{3}{4}=1\frac{2}{4}$ $1\frac{2}{4}+\frac{3}{4}=2\frac{1}{4}$ $2\frac{1}{4}+\frac{3}{4}=3$ $3+\frac{3}{4}=3\frac{3}{4}$ $3\frac{3}{4}+\frac{3}{4}=4\frac{2}{4}$ $4\frac{2}{4}+\frac{3}{4}=5\frac{1}{4}$ $5\frac{1}{4}+\frac{3}{4}=6$ 6 yards OR $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{24}{4}$ or 6 yards • Grouping and Combining Strategies: $\frac{3}{4}+\frac{3}{4}=1\frac{1}{2}$ (2 bows) $1\frac{1}{2}+1\frac{1}{2}=3$ (4 bows) 3 + 3 = 6 (8 bows) • Multiplicative Strategies: 4 groups of $\frac{3}{4}$ is 3, so 8 groups of $\frac{3}{4}$ is 6 yards OR 8 x $\frac{3}{4}$ = 8 x (3 x $\frac{1}{4}$) = (8 x 3) x $\frac{1}{4}$ or $\frac{24}{4}$ = 6 yards. This student is able to explain their thinking rather than just explaining the steps to the standard algorithm.
2. Number Lines *Iowa Core Mathematics* specifically states students in grades 3-6 should use the number line diagram. Standards addressing the number line are in both the Number and Operations – Fractions domain and the Measurement and Data domain. Have participants read the standards marked with an asterisk (*) from "Iowa Core Mathematics Measurement and Data Standards" (handout) and note the connections to fractions, number lines, line plots, or measurement scales. Discuss the following questions: • Where do fraction concepts appear in the Measurement and Data standards? How do the Measurement and Data standards support the development of fraction concepts? • How might students use a number line to solve the ribbon problem? • If no student uses a number line model to solve the ribbon problem, how might you introduce the number line as a possible strategy for this problem?	**2. Number Lines** We are looking at the Measurement and Data (MD) standards so teachers recognize the connections between the MD standards and the Number and Operations - Fractions standards. We want to show the number line is frequently used in the MD standards. Teachers should recognize they can use the MD standards to provide contextual situations to support the understanding of the number line diagram in the fraction standards. • Where do fraction concepts appear in the Measurement and Data standards? How do the Measurement and Data standards support the development of fraction concepts? The MD standards are briefly summarized below. For exact wording see *Iowa Core Mathematics*. The number line diagram is not new in third grade as students use the number line in second grade with whole numbers. The first two standards below refer to the number line in second grade. ○ 2.MD.B.6 (represent whole numbers and whole number sums and differences on a number line) ○ 2.MD.D.9 (measure to the nearest whole unit; make a line plot to the nearest whole unit)

	○ 3.MD.A.1 (measure time to nearest minute and solve word problems with a number line diagram) ○ 3.MD.A.2 (measure liquid volume using a measurement scale) ○ 3.MD.B.4 (measure with a ruler marked in halves and fourths; make line plot with halves and fourths) ○ 4.MD.A.1 (sizes of measurements within a system, 1 cm = 1/100 meter) ○ 4.MD.A.2 (solve word problems with fractions and decimals; represent measurements using number lines) ○ 4.MD.A.3 (apply perimeter formulas to rectangles – may have fractional side lengths) ○ 4.MD.B.4 (make line plots in halves, fourths, and eighths; solve fraction word problems using information in line plots) ○ 4.MD.C.5a (angle is 1/360 of a circle) ○ 5.MD.B.2 (make line plots in fractions of a unit) ○ 6.G.A.1 (find area of shapes – may have fractional side lengths) ○ 6.G.A.2 (find volume of rectangular prisms with fractional edge lengths) • How might students use a number line to solve the ribbon problem? Students might draw a number line to directly model the problem and count the groups of ¾. Discuss how this problem connects to standards 3.NF.A.2. and 4.MD.A.2.  • If no student uses a number line model to solve the ribbon problem, how might you introduce number line as a possible strategy for this problem? You might do one of the following: ○ Present the number line strategy and ask students to explain whether or not it is correct. ○ Show the tape diagram and number line diagram and ask students to explain how the two are connected.

	 ○ Ask students to confirm their answers by drawing a number line to represent the entire amount of ribbon and where each cut would be made.
3. Walking Problem Sam walks 9 miles each week. He walks $1\frac{1}{2}$ miles a day. How many days does he walk each week? Ask each small group to solve this problem using each of the strategies listed on page 62 of *Extending Children's Mathematics* or assign one strategy listed on page 62 to each group. Have groups show their thinking on an unlined sheet of paper using a black marker. Have each group share at least one strategy with the entire class. Ask participants to predict whether the ribbon problem or walking problem tends to be easier for students.	**3. Walking Problem** • If time is short, omit this problem or assign it as homework. • The walking problem is a Multiple Groups, measurement division problem and lends itself to a number line model. • The ribbon problem and walking problem are two different problem types, but can be solved with the same strategies. • The walking problem tends to be easier for students than the ribbon problem. This is due to the number choice. Most students find it easier to work with $\frac{1}{2}$ than $\frac{3}{4}$.

Session 3 Activity 5
Assignment

Approximate Time: 10 minutes
Materials:
- Handout "Session 3 Assignment Sheet"

Activity Description	Key Discussion Points
1. Read *Extending Children's Mathematics*: • Chapter 3 (pp. 48-68) 2. Discussion Question: Write out answers to the following question, so you are prepared to discuss your thoughts during session 4: • Which *Iowa Core Mathematics* standards might be developed with Multiple Groups problems? Explain your thinking. 3. Implementation Assignment 3: • Pose the ribbon problem, the walking problem, or both to your students without providing instruction on how to solve it. Read the problem with your students regardless of their grade level.	This assignment is similar to past assignments, but focuses on Multiple Groups problems for multiplication and measurement division.

• Classify your students' work according to the strategies on p. 62 of *Extending Children's Mathematics*. • Bring your students' work with you to Session 4. You will have time to share student work from only one problem.	